**Abstract**

We based on variational method to derive analytical expressions to describe the free expansion of a multi-charged condensate with a central vortex.

The evolution of the vortex core size and the asymptotic velocity during free expansion was studied with the cloud released from different harmonic trap configurations.

We found that an oblate condensate shape magnifying the effects of the vorticity in the expansion of the cloud. Besides that, the asymptotic velocity field of the cloud dimensions gave direct information about the interaction and circulation (\*), being also an important parameter to track.

**VI Results and discussion**

We solved eq.(18), starting with the equilibrium configuration inside the trap, to obtain the evolution of the parameters of the condensate cloud during the free expansion. We used the frequency *(…)* and expansion time *(…)* according with our experiments with Rubidium 87 alkaline atoms.

To calculate the initial condition we use (18) with the acceleration equal to zero or minimize the energy function () in relation of the widths ri *(reference of the appropriate equations showed in the previous section)*.

The graphic *#x* shows the initial energy of the system for a fixed interaction parameter as function of the trap geometry *(prolate –isotropic-oblate /* γ *(dimensionless) = 800 that correspond to real values in the experiment ~ as = …. Number atoms in condensate = … -sugestão de valores: 100 ao e 105)* for different circulations. The energy of the l =1 state differs very little from the fundamental l = 0 state, as it was expected in the Thomas Fermi regime (TF). About the higher order circulation states, this graphic shows an important physical effect that comes from the vortex circulation. The latter brings nor only an extra kinetic energy, but also affect the interaction energy since it spread the condensate volume due to the centrifugal effect.

In the following we will analyze the evolution of the core during the expansion. An important remark, however, is that our ansatz not describe the correct behave of the vortex core, since it will be constrained with the cloud dimensions. To improve such ansatz an additional parameter should be used to characterize the core expansion independently. Since we are in the TF regime, we used the relation between the vortex core and condensate central density to describe such evolution *(eq.#)*. In the graphic *#x* we represent the core equilibrium dimension together the approach proposed, based on the density at *pho=z=l=0*. It is clear that the two sizes coincide to l=1 but for higher order of angular momentum l the ansatz start to fail, mostly when we are close the oblate configuration, where the interaction energy is bigger – In this case our new characterization is even better – TF approach *(\*\*)*.

In the expansion (Figure 1), we compared the time to the aspect ratio inversion for different circulations, starting with each type of magnetic trap configuration.

(\*\*\*)

Using the value for the core *(#x)* and the solution for the evolution of the extension of the cloud given by eq. 18, we analyze how the vortex expand in relation to the cloud size. This analyze is particularly important since it contributes to improve the information that we extract from our absorption image technique made in time of flight; for example, in the classification and quantification of the number of vortex in the expanded cloud. The graphic of (Figure 5 \*\*\*\*) shows the evolution of such ration in time for a cloud with unitary circulation and starting with different trap anisotropy. The curves can be explained basing on the variation of the central density due to the trap configuration. For example, the initial core size diminished with the increase of λ since the central density increases. Otherwise the variation of the velocity of expansion in the axial direction has direct consequence in the dilution of the density in the vortex position, dictating its rate of expansion (5\*).

New paragraph (6\*)

New paragraph (7\*)

**General view**

The goal of this paper is bring analytical expression to derive the ratios between the core and cloud sizes. We use a gaussian ansatz to deal with the TF regime, to be able to extract simple expressions even for multi-charged vortex.

The results of the studied will be important to a better interpretation of our absorption image extracted in time of flight.

**OBSERVAÇÕES**

**(\*)** Acredito que podíamos trabalhar melhor essa parte, tentar analisar mais gráficos.

**(\*\*)** Acho que vc deveria mencionar em algum trecho exatamente a forma como vc variou o trap. Na verdade vc fixa wr e vai variando wz. Tem que ficar claro para o leitor que na configuração oblate vc the uma densidade central/potencial químico maior e portanto a aproximação TF ficaria melhor.

O Gráfico adicional que eu sugiro seria o tamanho do core como função do trap, para diferentes l’s. Para o ansatz sugeriria que vc tomasse a valor para o qual a densidade caísse para 1/e do valor central Sabe, acho que daria para vc montar analiticamente como se comportaria a razão: core ansatz/core approach como função do l respectivos, não?

**(\*\*\*)** reescrever as suas observações sobre a influência do aumento da circulação sobre o aspect ratio da nuvem, tendo como parâmetro de justificativa as analises a respeito da contribuição da energia cinética e de interação feitas no parágrafo anterior.

**(\*\*\*\*)** antiga Figura 1.d - sugiro que vc separe esse resultado dos anteriores

**(5\*)** Explique detalhadamente cada curva em particular, com base nessas observações.

Não esqueça de fazer especial menção ao nosso caso do experimento λ = 0.1 em que se observa que a taxa de expansão do core e raio da nuvem praticamente se mantém – acho que vc poderia estimar então o tamanho do core que teríamos para os nossos tempos de expansão. Daí a gente poderia inclusive colocar as nossas imagens no seu trabalho, e comparar com as suas previsões.

**(6\*)** Depois de colocar essa parte da comparação com os experimentos, gostaria de colocar também algumas simulações numéricas para comparar com seu variacional. Temos que ver se a variação do aspect ratio da nuvem com circulação alta está sendo bem caracterizado. Vou me organizar para poder fazer isso p vc, acho que é mais urgente que a precessão por enquanto.

Porém não descarto a possibilidade de vc fazer um paralelo com os outros artigos numéricos. Eles devem aparecer no seu texto em algum momento.

**(7\*)** Acho que vc poderia colocar no texto as expressões analíticas da velocidade assintótica e tentar com base nelas explicar os seus gráficos da Figura 2. Não sei, acho que vc poderia me fazer alguns gráficos com γ = 800 e só variando o trap, seguindo a tendência das analises anteriores.